Problem 1

1. For this question, we need to conform relation ‘≡’ contains reflectivity, symmetry and transitive on F.

For reflectivity:

Assume a ∈ F, because of a ≡ a, v(a) = v(a), so, (a, a) ∈ R. Therefore, reflectivity hold.

For symmetry:

Assume a ∈ F and b ∈ F. if (a, b) ∈ R, a ≡ b, then, v(a) = v(b),

And then, v(b) = v(a), then, (b, a)∈R

So, symmetry hold.

For transitive:

Assume a, b, c ∈ F, (a, b) ∈ R and (b, c) ∈ R,

Because of a ≡ b, b ≡ c, so, v(a) = v(b), v(b) = v(c), so, v(a) = v(c)

Therefore, (a, c) ∈ R and a ≡ c, so, transitive hold.

1. ⊥; ¬Τ; p ∧ ⊥; p ∧ ¬p

Because of ϕ ≡ ϕ’ and ψ ≡ ψ’

So, v(ϕ) = v(ϕ’) and v(ψ) = v(ψ’)

So, I should prove v(¬ϕ) = v(¬ϕ’)

For, v(¬ϕ) = v(¬ϕ’), we can get !v(ϕ) = !v(ϕ’),

Because v(ϕ) = v(ϕ’), so v(¬ϕ) = v(¬ϕ’) is true

So, ¬ϕ ≡ ¬ϕ’ is true.



v(ϕ ∧ ψ) = v(ϕ) && v(ψ)

v(ϕ’ ∧ ψ’) = v(ϕ’) && v(ψ’)

Because of ϕ ≡ ϕ’ and ψ ≡ ψ’

So, v(ϕ) = v(ϕ’) and v(ψ) = v(ψ’)

So, v(ϕ) && v(ψ) = v(ϕ’) && v(ψ’)

So, v(ϕ ∧ ψ) = v(ϕ’ ∧ ψ’)

So, ϕ ∧ ψ ≡ ϕ’ ∧ ψ’ is true.



v(ϕ ∨ ψ) = v(ϕ) || v(ψ)

v(ϕ’ ∨ ψ’) = v(ϕ’) || v(ψ’)

Because of ϕ ≡ ϕ’ and ψ ≡ ψ’

So, v(ϕ) = v(ϕ’) and v(ψ) = v(ψ’)

So, v(ϕ) || v(ψ) = v(ϕ’) || v(ψ’)

So, v(ϕ ∨ ψ) = v(ϕ’ ∨ ψ’)

So, ϕ ∨ ψ ≡ ϕ’ ∨ ψ’ is true.

1. From the complementation, we can know x ∨ x’ = 1; x ∧ x’ = 0.

So, [ϕ] ∨ [¬ϕ] = 1, [ϕ ∨ ¬ϕ] = [Τ], 1: [Τ]

And, [ϕ] ∧ [¬ϕ] = 0, [ϕ ∧ ¬ϕ] = [⊥], 0: [⊥]

From F≡, I get [x], [y], [z] equivalence classes.

For commutative:

Prove [x] ∨ [y] = [y] ∨ [x]

[x] ∨ [y] = [x ∨ y] (by define)

= [y ∨ x]

= [y] ∨ [x] (by define)

Prove [x] ∧ [y] = [x ∧ y]

[x] ∧ [y] = [x ∧ y] (by define)

= [y ∧ x]

= [y] ∧ [x] (by define)

For associative:

Prove [x] ∨ [y] ∨ [z] = [x] ∨ [y ∨ z]

[x] ∨ [y] ∨ [z] = [x] v [y ∨ z] is obvious true by define.

Prove [x] ∧ [y] ∧ [z] = [x] ∧ [y ∧ z]

[x] ∧ [y] ∧ [z] = [x] ∧ [y ∧ z] is obvious true by define.

For distributive:

Prove [x] ∨ ([y] ∧ [z]) = ([x] v [y]) ∧ ([x] ∨ [z])

[x] ∨ ([y] ∧ [z]) = [x] v [y ∧ z] (by define)

= [x ∨ (y ∧ z)] (by define)

= [(x ∨ y) ∧ (x ∨ z)] (by distributive)

= [x ∨ y] ∧ [x ∨ z] (by define)

= ([x] v [y]) ∧ ([x] ∨ [z]) (by define)

Prove [x] ∧ ([y] ∨ [z]) = ([x] ∧ [y]) ∨ ([x] ∧ [z])

[x] ∧ ([y] ∨ [z]) = [x] ∧ [y ∨ z] (by define)

= [x ∧ (y ∨z)] (by define)

= [(x ∧ y) ∨ (x ∧ z)] (by distributive)

= [x ∧ y] ∨ [x ∧ z] (by define)

= ([x] ∧ [y]) ∨ ([x] ∧ [z]) (by define)

For identity:

Prove [x] ∨ 0 = [x]

[x] ∨ 0 = [x ∨ 0] (by define)

= [x] (by identity)

Prove [x] ∧ 1 = [x]

[x] ∧ 1 = [x ∧ 1] (by define)

= [x] (by identity)

For complementation, I have proved it in the beginning.

Problem 2

1. In giving Petersen graph, every vertex is of degree 3, but for K5, every vertex is of degree 4, Therefore, the Petersen graph does not contain a subdivision of K5.
2. Every vertex is of degree 3 and there are 10 vertices.

The subgraph is as follow:

and then is:

5

6

9

4

0

1

8

7

2

3

and then is:

7

5

2

4

3

0

So, Petersen graph contains a subdivision of K3,3.

4

7

2

3

0

5

Problem 3

Vertices: Defence against the Dark Arts; Potions; Herbology; Transﬁguration; Charms.

Edges: (Defence against the Dark Arts, Potions);

(Defence against the Dark Arts, Charms);

(Potions, Herbology);

(Herbology, Transﬁguration);

(Transﬁguration, Charms).



For this question, I consider the color problem, the adjacent vertices cannot be the same color and ensure one color is the most.

1. To solve the problem:

Defence against the Dark Arts and Transﬁguration can be the same color, may be red

Herbology and Charms can be the same color, may be blue

Potions can be green color.

This make the one color be the most which is the red or blue color.

Problem 4

1. For this question, we have one node for the root, and then, for the left and right of tree, we have different number of nodes.

So, I have many ways: I set T(number of left nodes, number of right nodes)

So, T(n) = T(0, n-1) + T(1, n-2) + … + T(n-1, 0)

So, I have T(n) = T(0) \* T(n-1) + T(1) \* T(n-1) + … + T(n-1) \* T(0)

Simplify the T(n),

1. For a full binary tree, for every parent node, if they have child node, they should have two child nodes which is even number, and the root node is one, Therefore, the number of nodes is odd
2. For T(n), B(n)

I list out some result:

n T(n) B(n)

0 1 0

1 1 1

2 2 0

3 5 1

4 14 0

5 42 2

B(1) = T(0), B(3) = T(1), B(5) = T(2)

I image B(2n+1) = T(n)

1. The leaf node is n,

Then, from the assignment2, we can know the internal node is n-1

The all node is 2n-1

For every internal node, they have two choices (∧ or ∨)

For every leaf node, for the first, it has n choices, then, the second has n-1 choice, …

So, they have n! choices

Also, for every leaf node, they have two choice, which is ¬ or not ¬

So, they also have 2n choices.

So, T(n) = B(2n-1) \* n! \* 2n

Problem 5

1. For v1, there are v2 and v4 which can reach v1. For v2, v4, they have 1/3 possibility to reach to v1.

So, P1(n+1) = 1/3 \* P2(n) + 1/3 \* P4(n)

Similarly, P2(n+1) = 1/2 \* P1(n) + 1/3 \* P4(n) + 1/2 \* P3(n)

P3(n+1) = 1/3 \* P2(n) + 1/3 \* P4(n)

P4(n+1) = 1/2 \* P1(n) + 1/3 \* P2(n) + 1/2 \* P3(n)

1. From (a), we set Pi(n+1) = Pi(n),

So, we can get:

P1(n) = 1/3 \* P2(n) + 1/3 \* P4(n)

P2(n) = 1/2 \* P1(n) + 1/3 \* P4(n) + 1/2 \* P3(n)

P3(n) = 1/3 \* P2(n) + 1/3 \* P4(n)

P4(n) = 1/2 \* P1(n) + 1/3 \* P2(n) + 1/2 \* P3(n)

And we can simplify these equations to:

P2(n) = 3/2 \* P1(n)

P3(n) = P1(n)

P4(n) = 3/2 \* P1(n)

Because of P1(n) + P2(n) + P3(n) + P4(n) = 1,

So, we can get P1(n) = 1/5

P2(n) = 3/10

P3(n) = 1/5

P4(n) = 3/10

1. distance v1 to v1: 0

distance v1 to v2: 1

distance v1 to v3: 2

distance v1 to v4: 1

So, excepted distance = 0 \* P1(n) + 1 \* P2(n) + 2 \* P3(n) + 1 \* P4(n) = 1